

DEPARTMENT OF MATHEMATICS, NORTHEASTERN UNIVERSITY

MATH 7382 — TOPICS IN PROBABILITY / REPRESENTATION THEORY
OF BIG GROUPS AND PROBABILITY

FALL 2012

Instructor: Leonid Petrov. **Office:** 433 Lake Hall. **Phone:** 617-373-3893

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Class: T, F 5:00-6:30pm in 544 Nightingale Hall

Course webpage: http://www.math.neu.edu/~lpetrov/f12_neu_7382.html

Office hours: T,W,F from 2:00 to 3:00 pm. Alternative times: drop in, or make an appointment by email.

Course description:

In the course we will discuss probabilistic models which have a nice algebraic/combinatorial structure. Such structures are present in various areas and were first observed in the theory of random matrices in the 1960s. These fine properties of probabilistic models allow to extract very deep information about the behaviour of random objects under consideration. Along the way we will also discuss motivating constructions from representation theory (the theory of characters of ‘big’ groups) and necessary algebraic/combinatorial technique (most importantly, symmetric functions and determinantal point processes).

The course is intended as an introduction to a rapidly developing area which contains many interesting unsolved problems.

Grading: The grade will be based on graded problem sets.

Class Schedule (approximate):

1	Sep. 7	Introduction
2	Sep. 11 & 13	Algebraic structures in probabilistic models. De Finetti’s theorem.
3	Sep. 18 & 20	Facts from representation theory.
4	Sep. 25 & 27	Symmetric functions. Characters of unitary groups.
5	Oct. 2 & 4	Infinite-dimensional unitary group $U(\infty)$ and its characters.
6	Oct. 9 & 11 *)	Branching graphs. Examples. Gelfand-Tsetlin graph.
7	Oct. 16 & 18	Boundary of the Gelfand-Tsetlin graph and characters of $U(\infty)$.
8	Oct. 23 & 25	Totally nonnegative matrices, Gessel-Viennot determinants.
9	Oct. 30 & Nov. 1	Determinantal point processes.
10	Nov. 6 & 8	Determinantal point processes (cont.).
11	Nov. 13 & 15	Examples of determinantal processes.
12	Nov. 20 **)	Random lozenge tilings. Dimer coverings.
13	Nov. 27 & 29	Determinantal structure of random tilings.
14	Dec. 7	Playing with random tilings.

*) Oct. 12 — Surprise!

**) Nov. 23 — Thanksgiving recess, no class

Books: There is no single textbook. Lecture notes (in some form) will be posted, and relevant links to research papers (most of which are freely available at arXiv.org) will be provided.

More material can be found in the following books:

[1] I.G. Macdonald. Symmetric functions and Hall polynomials. Oxford University Press, 1995.

[2] W. Fulton, J. Harris. Representation Theory: A First Course. Springer, 1991.

It is absolutely not required that you purchase any of these books in order to succeed in the course.